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**Pedagogy and Subject-Specific  
Methodology for Mathematics  
Teachers**

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Die Hochschullehrgänge “Pädagogik und Fachdidaktik für LehrerInnen” (PFL) sind interdisziplinäre Lehrerfortbildungsprogramme der Abteilung “Schule und gesellschaftliches Lernen” des IFF. Die Durchführung der Lehrgänge erfolgt mit Unterstützung von BMUKA und BMWVK.

# **Pedagogy and Subject-Specific Methodology for Mathematics Teachers**

## **Reflections on an In-Service Education University Programme in Austria**

### *Inhaltsverzeichnis:*

1. Some remarks on the genesis and philosophy of the PFL-programme	1
2. PFL-mathematics - a little bit more concrete	3
The structure of PFL-mathematics 1991-93	3
PFL activities related to the guiding principles	4
3. Activities of teachers after a PFL course	6
References and selected literature on PFL-mathematics	6
Authors	7
Appendix	

# Pedagogy and Subject-Specific Methodology for Mathematics Teachers

## Reflections on an In-Service Education University Programme in Austria

*This article is about the two-year university course “PFL-mathematics” in Austria which aims at the (further) education of reflective practitioners who contribute to pedagogical and didactic innovations in classrooms and schools, especially with regard to mathematics instruction. Using some guiding principles of the PFL-programme, the article describes the structure and philosophy of the mathematics course and highlights activities of participants and team members.*

### 1. Some remarks on the genesis and philosophy of the PFL-programme

In the 1970's more and more high school teachers in Austria felt that in addition to their competence in two subjects (e.g. mathematics and geography) further development of their pedagogical and didactic competence would be required for coping successfully with the complexities of teaching and learning. In awareness of this need, an interdisciplinary team of researchers from the University of Klagenfurt began to plan the teacher in-service courses “*Pedagogy and Subject-Specific Methodology for Teachers*” (PFL is the German abbreviation for “Pädagogik und Fachdidaktik für LehrerInnen”) in the subjects English, German, history and mathematics. After a trial run in 1982 - 84 three PFL courses (for mathematics, English and three years later also for German) were institutionalised as university courses.

They are organized by the *Institute for Interdisciplinary Research and Continuing Education (IFF)* of the Austrian Universities of Innsbruck, Klagenfurt and Vienna, in cooperation with some departments of the University of Klagenfurt and regional INSET institutes and are supported by the Ministry of Science, Traffic and Arts' (BMWVK) and the Ministry for Education and Cultural Affairs (BMUKA).

Some organizational remarks: Each course lasts for four semesters and is attended by about 30 teachers. Participants are expected to be present at all meetings (e.g. two one-week seminars, two half-week seminars, four one and a half-day regional group meetings) and are required to write case studies on innovations introduced in their schools. After conclusion of the course participants receive a university certificate with a description of their achievements during the course. Most PFL courses are led by a team of 5 - 6 members (educationalists, subject-matter specialists, practitioners) who are responsible for the preparation and realization of the course and for follow-up activities (e.g. evaluation), all in all three years of intensive theoretical and practical work. The activities of the team members are seen not only as a contribution to the further education of teachers but also as an experience in interdisciplinary cooperation.

The *guiding principles of the PFL-programme* can be described as follows:

- One of the most important principles of PFL is stressing the importance and *interconnectedness of pedagogical and didactic aspects* of teaching and learning. Above all, the complexity

of the teacher's task cannot be reduced to content-related considerations. In mathematics instruction, for example, even topics like "proof" involve student motivation, different heuristic strategies, reflections on the nature of proof, discussions on students' understanding of proofs etc.

- In most cases the *starting point* of work within the PFL courses is the *practical experiences of the participants* in order to meet them "where they are", identifying strengths rather than weaknesses.
- *Action research* (see e.g. Altrichter/Posch/Somekh 1993), understood as *the systematic reflection of practitioners on action* (i.e. their professional activities in order to improve them), is used as a framework to achieve a broader situative understanding and to improve the quality of teaching. Within PFL courses the participants are required to do research work and to write – on the average – two case studies on self selected issues in which they have professional developmental interests.
- Communication among teachers often happens "in passing" and is often felt to be unsatisfactory. An attempt is made to find useful ways towards a *professional exchange of knowledge*, thus promoting the culture of communication on educational issues. *Communication and cooperation* among teachers is seen as a more and more demanding element of teachers' work. Countless good ideas of teachers exist only in their minds and are therefore not accessible to others. It is an important intention of the programme to make such private ideas public.
- PFL aims at providing opportunities to *connect individual and social learning experiences*. The open atmosphere in PFL courses is used as a basis for initiating and organizing communication and cooperation with colleagues, something which is in many cases difficult to achieve in a teacher's own school. The regional groups are small "professional communities" in which mutual understanding and constructive criticism are conducive processes.
- Promoting the *further development of theory and practice* implies a close cooperation between team members and participants, the former involving themselves in concrete and specific situations of the teachers' practical work, and the latter involving themselves in theoretical and general considerations. For this purpose it is an advantage if people come from different systems, namely "school" and "university", and are able to overcome their restricted perspectives.
- *Promoting active learning processes and reflecting on them* is a basic strategy in PFL courses in a double sense: firstly with regard to an epistemological understanding of learning, which sees the *learner as a producer* (and not consumer) of knowledge and secondly with regard to the conviction that *transfer from the course to the classroom* – where students also should be seen as producers – is more successful if the participants learn such processes through experience. "What can teachers learn from learners?" is a basic question within the courses.
- One aim of the programme is to motivate and qualify the participants *to organize further education for themselves* and for other teachers after conclusion of the course. Thus it is important that the participants be actively involved in planning and realization of the course, and increasingly take charge of their own further education within (and later outside) the course. Therefore the role of the team members within PFL develops – roughly speaking – from providing input and structure (to initiate teachers' activities) to organizing communication among the teachers as experts.

Each PFL course aims at contributing to each of the dimensions sketched above. Its strengths, however, lies in the whole composition of the course, decisively influenced by the people working in it – the team members as well as the teachers.

## 2. PFL-mathematics – a little bit more concrete

Although each of the three PFL-mathematics university courses (1985-87, 1988-90, 1991-93) had its own design, always being a further development based on the experiences and results of the former one, there is a kind of common philosophy as sketched in the guiding principles above.

In the following four of these guiding principles will be discussed more concretely and illustrated by selected PFL activities. It must be added that this report naturally lays emphasis on the last course the structure of which will be described below:

### The structure of PFL-mathematics 1991-93

The last PFL course was held especially for mathematics teachers of technical or commercial secondary schools; it was led by a team of six members and attended by 23 teachers.

At the beginning, the participants were asked for some individual preliminary work: They brought to the first meeting (Seminar I) a written description and documentation of a typical learning situation of their mathematics instruction.

*Seminar I*, lasting one week, gave an introduction to the aims, ideas and methods of “action research” and provided the opportunity for practical experience with some of these methods (e.g. analytic discourse, interview techniques). In addition, selected perspectives of education (especially mathematics education) were pointed out, three regional groups (each consisting of about eight teachers and two team members) were constituted, and further work in these regional groups was planned. Finally, ideas for (further) research work by participants (case studies), often related to their preliminary work, were discussed.

*Seminar II*, lasting three days, concentrated on concrete didactic and methodological work: Three parallel workshops dealt with Exploratory Data Analysis (EDA), assessment in mathematics instruction, and Algebra in the ninth school year respectively, and a lecture was given on the topic “Mathematics as a Tool for Presentation and Communication”.

In *Seminar III*, lasting one week, participants talked about their research work (first case studies). Next the role of computers in mathematics instruction and the role of applied mathematics were discussed. Thus this seminar dealt mainly with more general problems, basic ideas and the meaning of mathematics instruction.

*Seminar IV*, lasting three days, was organized by the participants themselves to a considerable extent. Participants presented their research work (their second case studies) and in (parallel) workshops they analyzed didactic aspects of handling functions, problems of constructing a questionnaire, and problems of data manipulation.

The first of a total of four regional group meetings were held after Seminar I, the second between Seminar II and Seminar III, and the last two regional group meetings were held between Seminar III and Seminar IV. At these regional group meetings discussions of the case studies and of specialized literature were predominant.

The contents of this PFL-mathematics course 1991-93 and its organization are described schematically in the appendix.

## PFL activities related to the guiding principles

PFL courses try to consider *pedagogical aspects* as well as *didactic aspects* – and especially their *interconnectedness*. A consequence is illustrated in the following statement of a participant (1988-90):

*“The personal relationship to my pupils is now more important than before; my thoughts do not center on subject matter alone”.*

In the following some examples from seminars of the last PFL-mathematics are given: The lecture “The Meaning of Mathematics Instruction” analyzed, from different points of view, in which ways mathematics instruction (and what type of mathematical instruction) could be meaningful for pupils as well as for teachers and for human society.

In the unit “Computers in Mathematics Instruction” not only technological or methodological aspects were discussed, but also pedagogical (e.g. teaching methods, motivation) and psychological aspects (e.g. changes in the processes of thinking and learning).

In the workshop on “Assessments in Mathematics Instruction” the close connection between (general) aims of mathematical teaching, teaching methods, assessment, pupils’ motivation were discussed.

This interconnectedness of pedagogical and didactic aspects can also be found in most of the participants’ case studies, e.g. on problems of motivation in connection with mathematical proofs or applications, on previous mathematical knowledge and experience, and on effects of different forms of assessment.

In PFL-mathematics an attempt was made to promote *communication* and *cooperation* among teachers and to improve the culture of teaching mathematics.

Different forms of communication and cooperation (in plenary sessions, regional groups, subgroups, workshops, open staff etc.) were used and practiced. An important example is the “analytic discourse”.

The starting point of an analytic discourse is a *concrete, practical problem, presented by a participant to a group of colleagues*. The basic aim of an analytic discourse is to gain an in depth understanding of the situation of this teacher. For this purpose the group is asked to pose questions only. Neither criticism nor suggestions nor the participant’s own stories are accepted. There are good reasons for these rules, e.g.: If participants come in with their own suggestions or experiences attention is distracted from the situation of the reporting teacher; moreover, criticism tends to stimulate a defensive attitude and to contribute very little to gaining understanding. Thus it is only permitted to ask questions in order to understand the problem, to find out about as many aspects of the situation as possible and to go into the topic as deeply as possible. We consider the promotion of such (or similar) ways of analysis as important means for an improvement of communication among teachers.

Many initiatives and activities of PFL teachers showed that PFL can lead to more cooperation and better communication among teachers. We confine ourselves to quoting a participant (1988-1990):

*“I have also become braver and take more initiatives in my relationships with colleagues. Small contributions at staff meetings, even organizing parts of a staff meeting with colleagues, preparing and carrying out projects together, are activities I tackle because I have observed, seen and learned very much ...”*

But these effects are not restricted to the communication among teachers:  
*“If difficulties arise I now try to talk about them with my pupils and don’t solve the problem just by changing my teaching approach or making demands on the pupils.”*

PFL courses try to motivate and to encourage participants to do research work within the framework of “action research”. In this report we want to give a concrete example of the research of one teacher who participated in the last PFL-mathematics course:

Her starting point was her individual preliminary work in which she described and illustrated her dissatisfaction and her problems with the teaching of mathematical proofs. The description was rather unsystematic and vague. In the analytic discourse on Seminar I it was possible to clarify her starting point which mainly related to pupils’ understanding and motivation. After the discussion, the teacher decided to investigate this problem more closely. She elaborated her own point of view with regard to mathematical proofs, encouraged her pupils to tell her their opinions (in written form and anonymous), and discussed this problem with some of her colleagues (not only mathematics teachers) at school.

At the end, she came to several general and also specific pedagogical and didactic conclusions:

*“In the future, I’ll talk with my pupils much more about proofs ... I’ll confront them more often with problems, where not the numerical or algebraic solution is asked for, but the process of solution and the reasons for it must be described correctly ... and I’ll try to give them a more balanced view of all aspects of mathematics.”*

The teacher documented her experiences from her starting point to the final results in a paper of 26 pages. She made it available not only for the other participants of this PFL-mathematics but also for her colleagues at school.

Finally we want to illustrate the guiding principle *self-organization of further education*.

At Seminar I of the last PFL-mathematics course most of the input and the whole organization were the responsibility of the team members: team members provided the design of the seminar, they gave the lectures, they managed the analytic discourse and they led all discussions in the plenary sessions as well as in the regional groups. Only at the end of this seminar, some of the participants had to give a short presentation of the results of their interviews and the regional groups, led by team members, had to plan their first one-and-a-half day meeting.

At Seminar II the team members organized three workshops based already on specific interests of the participants, in which they increasingly organized their work on their own. In the regional groups, there was a similar transition of responsibility for input and organization from team members to participants.

At Seminar III most of the input was provided by the participants in the form of presentations on their research and demonstrations of computeruses in mathematics instruction. For the workshop “The Role of Applied Mathematics in Secondary Education” e.g., the participants produced considerable input, which was later analyzed and discussed in small groups of participants and team members led by participants.

Finally, Seminar IV was almost exclusively organized by the participants: some teachers presented their research work; others organized and led workshops; participants also gave evaluations of the entire course (presented on posters) and led the plenary session. Of course, the farewell evening was also managed by the participants.

It will be shown in the last section of this report, that the experiences and competences gained in the PFL course had significant consequences.

### 3. Activities of teachers after a PFL course

Feedback from former PFL participants shows that they have involved themselves in a number of individual and joint activities which contribute to pedagogical and didactic innovation in classrooms and schools. In many cases they act as “agents of change” in their region, are engaged in teacher in-service-courses or in teacher education, and actively participate in conferences in which innovative work by teachers is presented. One initiative of a group of PFL participants will be described as an example.

In the last seminar of the PFL-mathematics course 1988-90 a group of participants organized a workshop on the topic “Elements of an Alternative School”. The workshop included a broad spectrum of activities – from general considerations on educational goals to concrete reflections on possible changes in mathematics instruction and motivated some participants to continue to cooperate. One of them organized an excursion to Tuscany (Italy) where, among other activities two secondary schools were visited. Eight participants and two team members of the – in the meantime completed – PFL course took part, observed some Italian classroom instruction, held discussions with teachers and principals, and reflected on their observations and impressions.

Half a year later, the group met again to discuss “project work” in mathematics. Finally they decided to initiate meetings twice a year and after some time they felt that they – especially the loyal core of seven people – were more than an ordinary group of teachers. In a report about their work in our journal for PFL participants they explained their intentions and the name “AFL - Aktion Forschende LehrerInnen” they had given themselves:

*“AFL is an abbreviation for ‘Researching Teachers’ Initiative’. With this name we want to express on the one hand our mutual interest in action research, for which we have been inspired within the PFL course. On the other hand we want to express our interest in actions, in activities in the field of teacher in-service education. Thereby we start from the assumption that teachers have, or can acquire, enough competence to realize teacher in-service education by themselves, that nobody knows as much about school and learning as they themselves, that nobody knows their needs as well as they themselves. And we are not only interested in our own further education but also in putting the idea of ‘teachers train teachers’ into practice ...”*

One related activity was undertaking different interdisciplinary projects as joint research projects of teachers and pupils whereby one member of the group played the role of the project-manager (coming from the “outside”). At present the members of AFL are at work on a book(let) with the title “Hands-on Mathematics”.

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